Robust Rotation and Translation Estimation in Multiview Reconstruction

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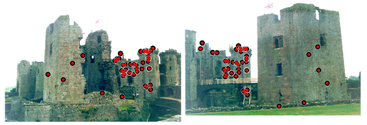
3D reconstruction from a difficult image set

- many images — wide & narrow base-line, panoramas
- repetitive structures, similar objects

⟶ mismatches satisfying epipolar geometries (EGs)



⟹ **mismatch removal**

⟶ non-existent EGs

⟹ **non-existent EG removal**

Problem formulation

Perspective camera projection: $\lambda^i_p \underbrace{\mathbf{x}^i_p}_{3\times1} = \underbrace{\mathtt{P}^i}_{3\times4}\underbrace{\mathtt{X}_p}_{4\times1}$

$$\underbrace{\begin{bmatrix} \lambda^1_1\mathbf{x}^1_1 & \lambda^1_2\mathbf{x}^1_2 & \dots & \lambda^1_n\mathbf{x}^1_n \\ \times & \lambda^2_2\mathbf{x}^2_2 & & \times \\ \vdots & & \ddots & \vdots \\ \lambda^m_1\mathbf{x}^m_1 & \times & \dots & \lambda^m_n\mathbf{x}^m_n \end{bmatrix}}_{\text{rescaled measurement matrix}} = \underbrace{\begin{bmatrix} \mathtt{P}^1 \\ \mathtt{P}^2 \\ \vdots \\ \mathtt{P}^m \end{bmatrix}}_{\substack{3m\times4 \\ \text{motion}}} \underbrace{\begin{bmatrix} \mathtt{X}_1 & \mathtt{X}_2 & \dots & \mathtt{X}_n \end{bmatrix}}_{\substack{4\times n \\ \text{structure}}}$$

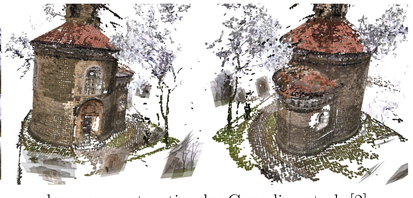
- 99.9% occlusions ⟹ imputation, damped Newton [1] with random initialization impossible

Solution: rotation and translation registration

$$\underbrace{\mathtt{P}^i}_{3\times4} = \underbrace{\mathtt{K}^i}_{3\times3}\underbrace{\mathtt{R}^i}_{3\times3}\underbrace{\begin{bmatrix}\mathtt{I}\end{bmatrix}}_{3\times3}\underbrace{\begin{bmatrix}-\mathtt{t}^i\end{bmatrix}}_{3\times1}$$

Precise results

124 images (24 shown)



dense reconstruction by Cornelius et al. [2]

Contribution 1: Rotation registration

Task *Given relative rotations [5],* $\mathtt{R}^{ij}$*, estimate registered rotations,* $\mathtt{R}^i$*, s.t. the relations among them are given by* $\mathtt{R}^{ij}$*:*

$$\mathtt{R}^j = \mathtt{R}^{ij}\mathtt{R}^i \quad \text{for all } ij \tag{1}$$

$$\mathtt{R}^i \text{ orthonormal for } i = 1,\dots,m \tag{2}$$

Solution without satisfying the orthonormality constraints (2)

1. System (1) consists of three smaller subsystems

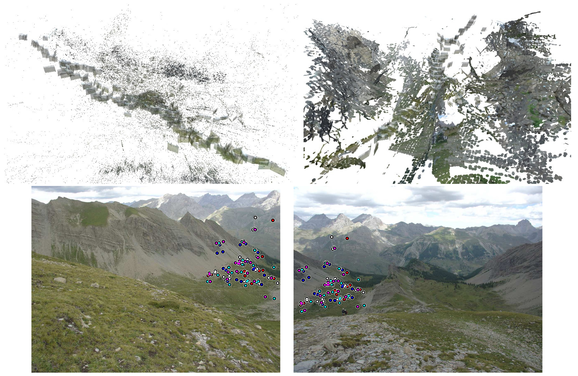
$$\mathbf{r}^j_k - \mathtt{R}^{ij}\mathbf{r}^i_k = 0_{3\times1} \quad \text{for all } ij \tag{3}$$

for $k = 1,2,3$, where $\mathbf{r}^i_k$ are columns of $\mathtt{R}^i$, $\mathtt{R}^i = [\mathbf{r}^i_1\mathbf{r}^i_2\mathbf{r}^i_3]$.

2. Get the best three linearly independent least square solutions to (3). — MATLAB's EIGS
3. Find the closest rotation to $\mathtt{R}^i$ in the Frobenius norm using SVD.

- global optimum, global error propagation
- more stable, faster and simpler than [4]

259 views using 2049 relative rotations in 0.37 sec



Contribution 2: Identifying mismatches satisfying an EG

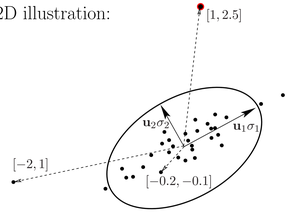
- translations are registered by minimizing the L_∞-norm (maximum) of the reprojection error [3] ⟹ all mismatches should be removed

Columns of $\underbrace{\begin{bmatrix} \lambda^1_1\mathbf{x}^1_1 & \lambda^1_2\mathbf{x}^1_2 & \dots \\ \lambda^2_1\mathbf{x}^2_1 & \lambda^2_2\mathbf{x}^2_2 & \dots \end{bmatrix}_{6\times n}}_{\mathtt{PX}}$ span *rescaled image space* (4D).

Observation *True matches between the same surface are (i) localized close to one another in the images and (ii) have similar depths.* ⟹ true matches form clusters in the rescaled image space while mismatches are far from the remaining data due to incorrect depths

- any clustering algorithm could be used but the following works:
- fit a Gaussian to the data in PX and remove 25% of most outlying points as the most likely mismatches

A 2D illustration:



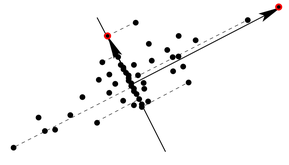
$[\mathbf{u}_1\mathbf{u}_2]\,\mathrm{diag}(\sigma_1,\sigma_2)\mathtt{V}^\top$ is the "economy size" SVD factorization of PX

true matches — four most different points

Reconstruction represented by four points

- after removing mismatches, choose four most different points:

1. Identify the most different column in PX by fitting the Gaussian.
2. Project PX onto the span of the column and subtract from PX.
3. Repeat four times.



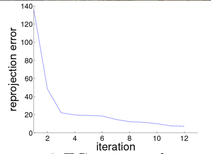
Contribution 3: Identifying non-existent EGs

- non-existent EGs deteriorate the quality of rotation registration (3) ⟹ non-existent EGs should be removed

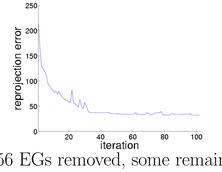
Proposition [6] *For a wide class of* L_∞ *problems, measurements with the greatest residual must contain at least one outlier.*

- here: least squares rotation estimate + L_∞ translation estimate
- in practice, points from non-existent EGs have large residua ⟹ remove such EGs

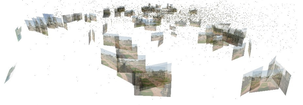
1. Register rotations (3) and translations [3].
2. If all residua are small, break.
3. Remove EGs in which some of the four points reached the maximum residual and go to 1.



13 EGs removed



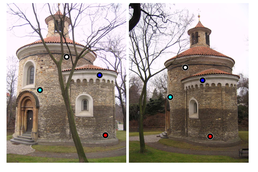
156 EGs removed, some remain



⟶ maximum error 98 ⟶ 22 pxl

speedup of factor 2000

the chosen points lie in different depths and thus capture the 3D geometry of the image pair well



References

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