3D Reconstruction by Fitting Low-rank Matrices with Missing Data

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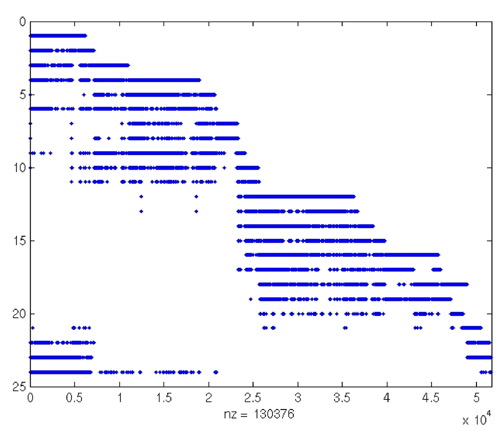
3D reconstruction from a difficult image set

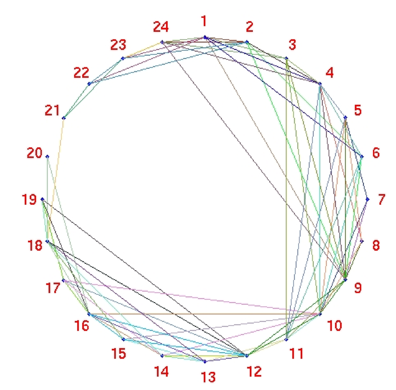
- wide & narrow base-line
- image scaling, varying focal length
- many points seen in few images & no one in all

Dense reconstruction by Cornelius et al. [1]

cameras



89% data missing



Epipolar geometries

Problem Formulation

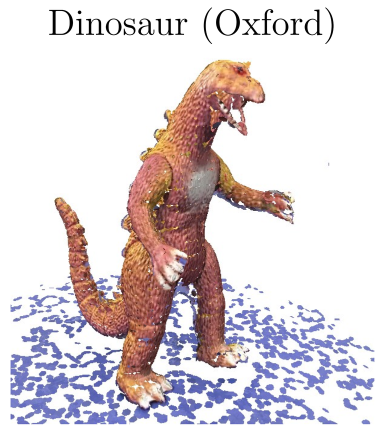
Perspective camera projection: $\underbrace{\lambda_p^i \mathbf{x}_p^i}_{3\times 1} = \underbrace{\mathtt{P}^i}_{3\times 4}\underbrace{\mathtt{X}_p}_{4\times 1}$

$$\underbrace{\begin{bmatrix} \lambda_1^1\mathbf{x}_1^1 & \lambda_2^1\mathbf{x}_2^1 & \dots & \lambda_n^1\mathbf{x}_n^1 \\ \times & \lambda_2^2\mathbf{x}_2^2 & & \times \\ \vdots & & \ddots & \vdots \\ \lambda_1^m\mathbf{x}_1^m & \times & \dots & \lambda_n^m\mathbf{x}_n^m \end{bmatrix}}_{\mathbf{y}} = \underbrace{\begin{bmatrix} \mathtt{P}^1 \\ \mathtt{P}^2 \\ \vdots \\ \mathtt{P}^m \end{bmatrix}}_{3m\times 4} \underbrace{\begin{bmatrix} \mathtt{X}_1 \, \mathtt{X}_2 \dots \mathtt{X}_n \end{bmatrix}}_{4\times n}$$

rescaled measurement matrix — motion — structure

More results

Dinosaur (Oxford)

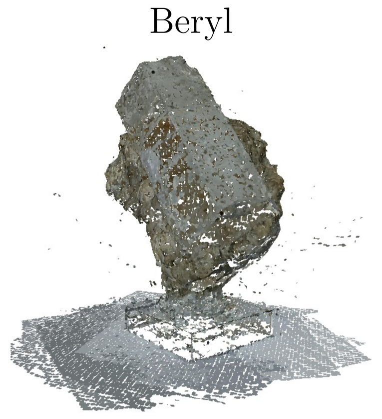


36 img [720x576]
Harris operator
90.84% occlusions

Head

10 images [1391x1043]
distinguished regions (DR)
73% occlusions

Beryl



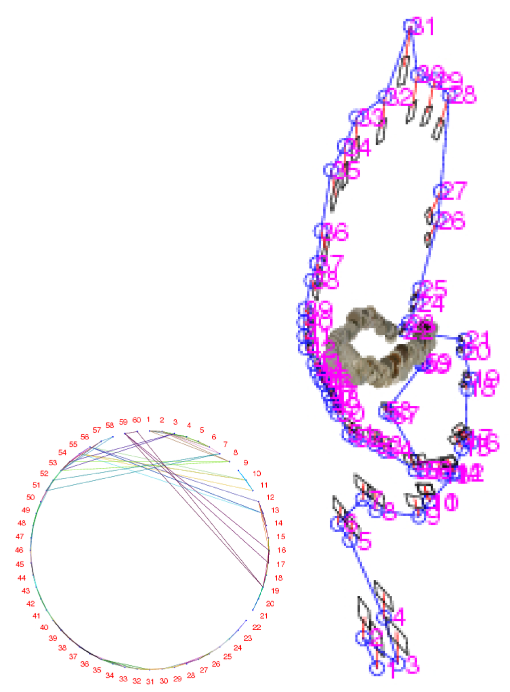
64 images [811x1219]
DR
94% occlusions

St. George rotunda

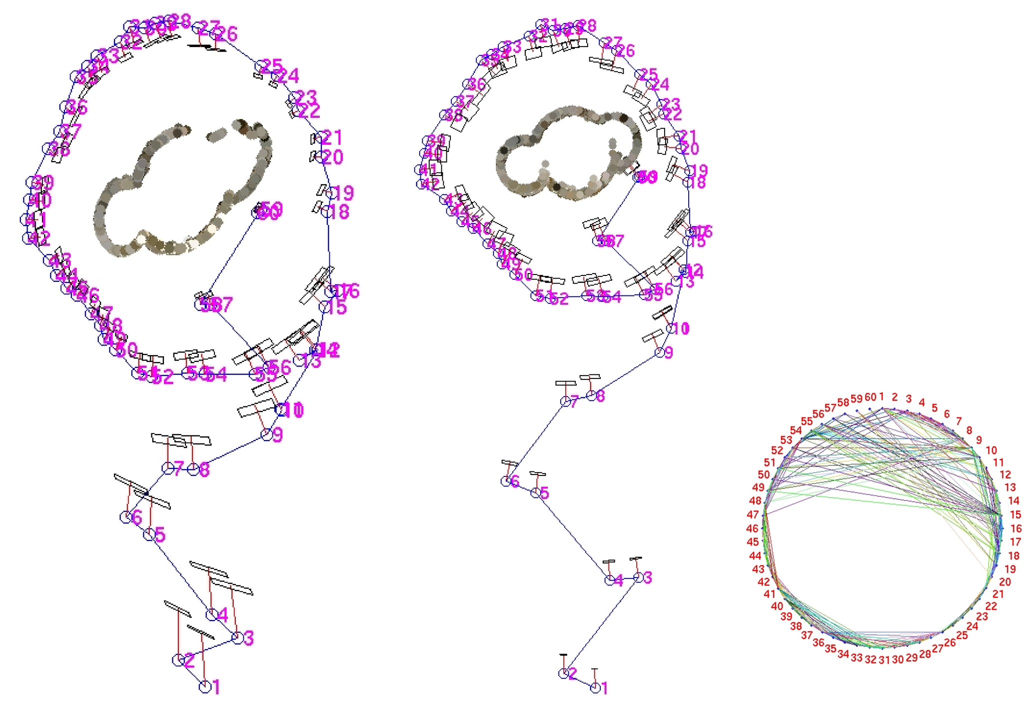
60 images [1100x1650]
DR
92% occlusions

Contribution 1: Computing depths λ_p^i

- linear formulation for computing consistent λ_p^i
- all λ_p^i in one system $\Longrightarrow$ stability, global error propagation



minimal 58 triples



166 triples

Independent reconstructions from image pairs computed from the fundamental matrix $\mathtt{F}^{ij}$:

$$\begin{bmatrix} \gamma_1^i\mathbf{x}_{p_1}^i \cdots \gamma_z^i\mathbf{x}_{p_z}^i \\ \gamma_1^j\mathbf{x}_{p_1}^j \cdots \gamma_z^j\mathbf{x}_{p_z}^j \end{bmatrix} = \begin{bmatrix} \hat{\mathtt{P}}^i \\ \hat{\mathtt{P}}^j \end{bmatrix} \begin{bmatrix} \hat{\mathtt{X}}_1 \dots \hat{\mathtt{X}}_z \end{bmatrix}$$

There exist consistent λ_p^i such that

$$\begin{bmatrix} r^{ij}[\lambda_1^i \dots \lambda_z^i] \\ s^{ij}[\lambda_1^j \dots \lambda_z^j] \end{bmatrix} = \begin{bmatrix} c_1^{ij}\begin{pmatrix}\gamma_1^i \\ \gamma_1^j\end{pmatrix} & \dots & c_z^{ij}\begin{pmatrix}\gamma_z^i \\ \gamma_z^j\end{pmatrix} \end{bmatrix}$$

$$\Downarrow$$

$$r^{ij}/s^{ij}\begin{bmatrix} \frac{\lambda_1^i}{\lambda_1^j} & \dots & \frac{\lambda_z^i}{\lambda_z^j} \end{bmatrix} = \begin{bmatrix} \frac{\gamma_1^i}{\gamma_1^j} & \dots & \frac{\gamma_z^i}{\gamma_z^j} \end{bmatrix}$$

$$\Downarrow$$

$$\alpha^{ij}\begin{bmatrix} \lambda_1^i & \dots & \lambda_z^i \end{bmatrix} = \begin{bmatrix} g_1^{ij}\lambda_1^j & \dots & g_z^{ij}\lambda_z^j \end{bmatrix}$$ / cheirality → all positive → logarithm

$$\overline{\alpha}^{ij} + \begin{bmatrix} \overline{\lambda}_{p_1}^i & \dots & \overline{\lambda}_{p_z}^i \end{bmatrix} = \begin{bmatrix} \overline{g}_1^{ij} + \overline{\lambda}_{p_1}^j & \dots & \overline{g}_z^{ij} + \overline{\lambda}_{p_z}^j \end{bmatrix}$$

$$\left.\begin{aligned} \overline{\alpha}^{ij} + \overline{\lambda}_{p_1}^i - \overline{\lambda}_{p_1}^j &= \overline{g}_1^{ij} \\ &\vdots \\ \overline{\alpha}^{ij} + \overline{\lambda}_{p_z}^i - \overline{\lambda}_{p_z}^j &= \overline{g}_z^{ij} \end{aligned}\right\} \quad (1)$$

sparse system of linear equations

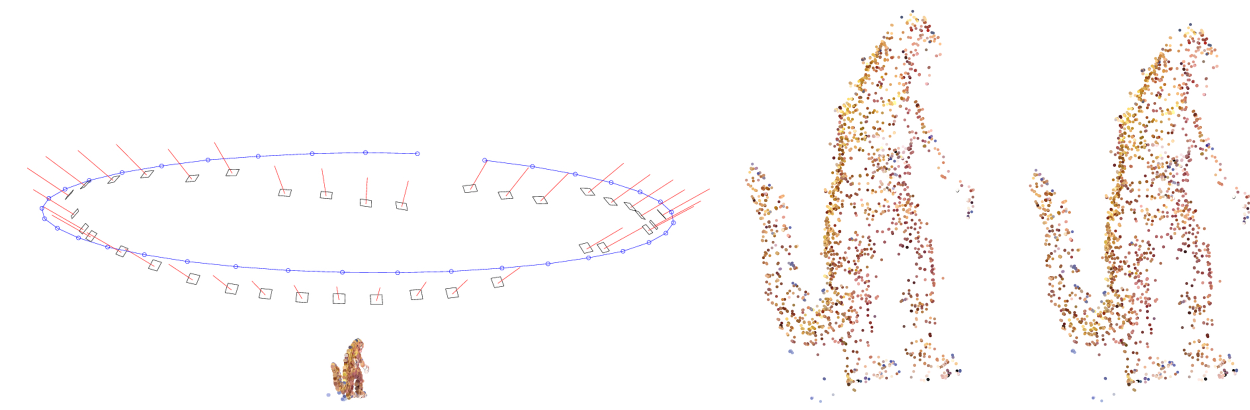
References

[1] Hugo Cornelius, Radim Šára, Daniel Martinec, Tomáš Pajdla, Ondřej Chum, and Jiří Matas. Towards complete free-form reconstruction of complex 3D scenes from an unordered set of uncalibrated images. In D. Comaniciu, R. Mester, and K. Kanatani, editors, *Proc ECCV Workshop Statistical Methods in Video Processing*, volume LNCS 3247, pages 1–12, Heidelberg, Germany, May 2004. Springer-Verlag.

Contribution 2: Computing the best rank-4 approximation

$$\mathbf{y} = \mathtt{PX}$$

- linear reformulation of the bilinear problem
- it is a good approximation of the original problem
- global error propagation



tracks via triples — 0.25 pxl — BA: 0.24 pxl

$\exists \mathtt{P}^\mathbf{i}, \mathtt{X}_\mathbf{p}$: $\mathbf{y}_\mathbf{p}^\mathbf{i} = \mathtt{P}^\mathbf{i}\mathtt{X}_\mathbf{p}$ e.g. $\mathbf{i} = \{1,2\}, \mathbf{p} = \{1,2,3,4\}$

If $\mathbf{i}$, $\mathbf{p}$ such that rank $\mathbf{y}_\mathbf{p}^\mathbf{i} = 4 \Longrightarrow$ rank $\mathtt{P}^\mathbf{i}, \mathtt{X}_\mathbf{p} = 4$

Moreover, if length($\mathbf{p}$) $= 4 \Longrightarrow \exists \mathtt{X}_\mathbf{p}^{-1}$ and

$$\mathbf{y}_\mathbf{p}^\mathbf{i} = \mathtt{P}^\mathbf{i}\mathtt{X}_\mathbf{p} \iff \mathbf{y}_\mathbf{p}^\mathbf{i}\mathtt{X}_\mathbf{p}^{-1} = \mathtt{P}^\mathbf{i}$$

bilinear problem in $\mathtt{P}^\mathbf{i}$, $\mathtt{X}_\mathbf{p}$ — linear problem in $\mathtt{P}^\mathbf{i}$, $\mathtt{X}_\mathbf{p}^{-1}$

Use multiple submatrices $\hat{\mathtt{P}}_t = \mathbf{y}_{\mathbf{p}_t}^{\mathbf{i}_t}$, $t = 1, \dots, T$ and solve

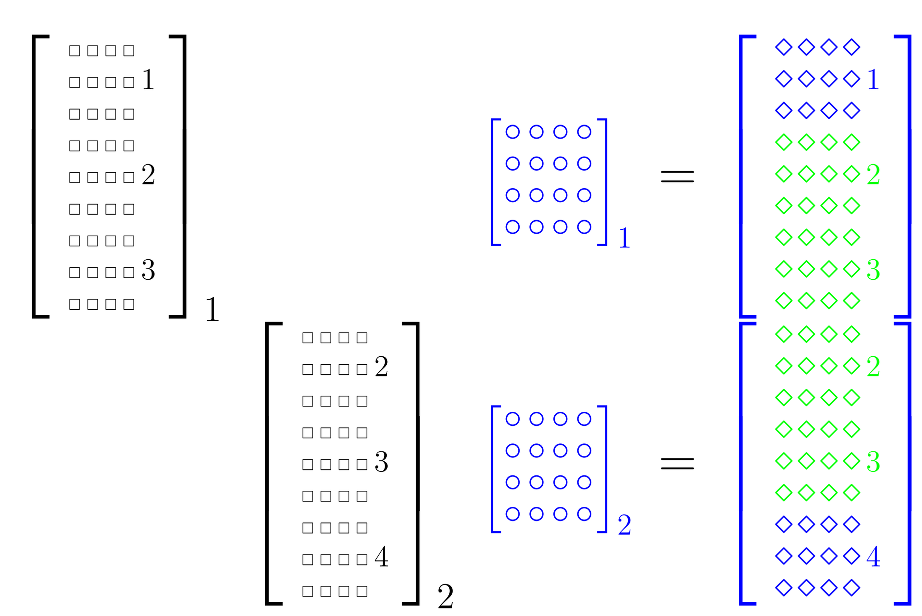
$$\left.\begin{aligned} \hat{\mathtt{P}}_1\mathtt{H}_1 &= \mathtt{P}^{\mathbf{i}_1} \\ &\vdots \\ \hat{\mathtt{P}}_T\mathtt{H}_T &= \mathtt{P}^{\mathbf{i}_T} \end{aligned}\right\} \quad (2)$$

sparse system of linear equations

Improvement: use an SVD-fit to many columns in $\mathbf{y}_{\mathbf{p}_t}^{\mathbf{i}_t}$ for image pairs and triples: $\hat{\mathtt{P}}_t = \mathtt{U}_{1,2,3,4}$ where $\mathbf{y}_{\mathbf{p}_t}^{\mathbf{i}_t} = \mathtt{U}\,\mathrm{diag}(\sigma_1, \dots, \sigma_z)\mathtt{V}^\top$

Example: Gluing two partial reconstructions

$$\hat{\mathtt{P}}_1\mathtt{H}_1 = \mathtt{P}^{\{1,2,3\}}$$
$$\hat{\mathtt{P}}_2\mathtt{H}_2 = \mathtt{P}^{\{2,3,4\}}$$



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