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Daniel Martinec and Tomáš Pajdla

{martid1, pajdla}@cmp.felk.cvut.cz

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Center for Machine Perception, Department of Cybernetics
Faculty of Electrical Engineering, Czech Technical University
Technická 2, 166 27 Prague 6, Czech Republic
fax +420 2 2435 7385, phone +420 2 2435 7637, www: http://cmp.felk.cvut.cz

Line Reconstruction from Many Perspective Images by Factorization

Daniel Martinec Tomáš Pajdla*
Center for Machine Perception, Dept. of Cybernetics, Faculty of Elec. Eng.
Czech Technical University in Prague, Karlovo nám. 13, 121 35 Prague, Czech Rep.
{martid1,pajdla}@cmp.felk.cvut.cz

Abstract

This paper brings a new method for line reconstruction from many perspective images by factorization of a matrix containing line correspondences. No point correspondences are used. We formulate the reconstruction from line correspondences in the language of Plücker line coordinates. The reconstruction is posed as the factorization of $3m \times n$ matrix $\mathtt{S}$ *into the product* $\mathtt{S} = \mathtt{QL}$ *of* $3m \times 6$ *projection matrix* $\mathtt{Q}$ *and* $6 \times n$ *line matrix* $\mathtt{L}$*, both satisfying Klein identities. The matrix* $\mathtt{S}$ *contains coordinates of lines detected in perspective images. Similarly to reconstruction from point correspondences in perspective images, the matrix* $\mathtt{S}$ *has to be properly rescaled before it can be factorized. We propose a scaling of image line coordinates based on trifocal tensors that is analogical to the scaling proposed by Sturm and Triggs for points. We propose an* SVD *based factorization enforcing Klein identities on* $\mathtt{Q}$ *and* $\mathtt{L}$ *in a noise-free situation. We show experiments on real data that suggest that a good reconstruction may be obtained even if data is noisy and the identities are not enforced exactly. We also discuss an extension of the method for images with occlusions.*

1. Introduction

Reconstruction from image point correspondences using factorization [11, 2] allows an elegant and uniform treatment of the reconstruction problem if more than four images of the scene are available. With more than four images, tensorial approach to the geometry of multiple cameras [2] becomes less elegant as there is no single algebraic equation that would contain coordinates from more than four views. In this paper we extend the reconstruction by factorization to line correspondences in many perspective images.

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Originally proposed for point correspondences in orthographic images by Tomasi and Kanade [11], the factorization has been later extended for point correspondences in perspective images by Sturm and Triggs [10]. Another line of research dealing with occlusions in orthographic images, i.e. missing data in the point measurement matrix, has been also started by Tomasi and Kanade [11] and further improved by Jacobs [4] who provided a valuable linearly-algebraic insight into the problem. Martinec and Pajdla [8] extended works of Sturm and Triggs and Jacobs and worked out the factorization from perspective images with occlusions, which can solve any image set that provides a trackable image point measurement matrix. The last missing bit of work has been added by Huynh and Heyden [3] who showed how to exploit RANSAC for robust completion of point measurement matrix for orthographic cameras. By extending the work by Martinec and Pajdla to deal with outliers [7], a completely general and robust reconstruction technique has been obtained for point correspondences in perspective images.

It is possible to reconstruct lines in space from line correspondences in images [2]. Line correspondences are attractive for a number of reasons. There are many lines in man-made environments. Lines can be sometimes detected more precisely than points. Lines are less affected by occlusions as projections of different parts of the same line can be used to reconstruct it.

Quan and Kanade [9] proposed a factorization based reconstruction of lines from affine images. They have used a line representation that allowed them in an orthographic setup to transform the problem of finding line directions into a problem of a point factorization from 1D projective camera. Kahl and Heyden [5] proposed factorization of points, lines and conics under affine projection. No attempt for addressing the problem in the perspective setup has been known up to now (see, e.g., [6]).

In this paper we formulate the reconstruction from line correspondences by factorization in a perspective setup. We concentrate on giving the formulation of the problem and on demonstrating in experiments that meaningful results are obtained. We show that a suitable formulation leads to a well posed problem that can be solved even in the presence

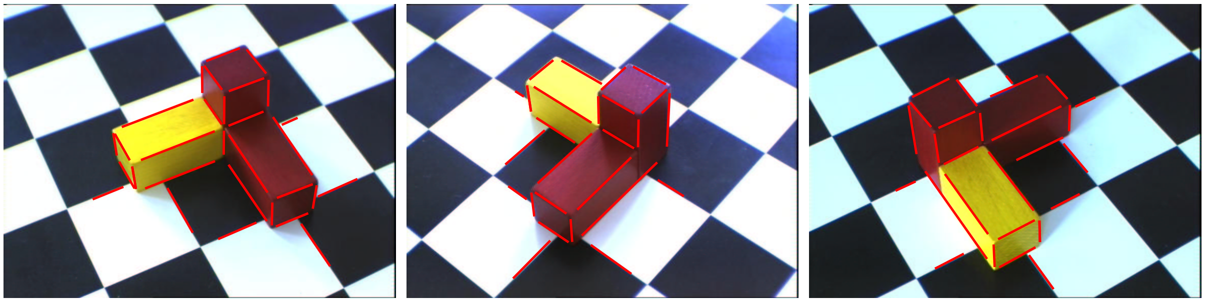


Figure 1: *Line correspondences.* Line correspondences can often be established even though line end-points are occluded

of noise without solving every step optimally in full generality. We concentrate here only on the situation when there is no occlusion in the scene. An extension towards occlusions is possible but will be presented elsewhere. Foundations for our method follow.

First, we use Plücker coordinates to represent lines in three dimensional space as well as in images. Plücker coordinates of lines in space are *linearly* projected to Plücker coordinates of lines in images, exactly the same way as homogeneous 3D point coordinates are projected linearly into homogeneous 2D point coordinates. Only thanks to the linearity of the projection, a factorization is possible.

Second, a scaling method has to be proposed in order to properly scale image line coordinates in the line measurement matrix. We propose a line scaling technique that exploits trifocal tensors of line correspondences across triplets of views.

Third, the representation of lines by Plücker coordinates requires that the elements of a six dimensional Plücker vector satisfy a quadratic identity to represent a line in space. Thus, it is not enough to factorize the line measurement matrix by a simple SVD-based factorization but it is necessary to do it so that the reconstructed representation of structure and motion satisfy all necessary identities. We show how to achieve it for a noise free data by a simple transformation of an SVD-based factorization into the coordinate system of a reconstruction from one triplet of images.

Finally, everything becomes more difficult when noise is present in data as it is more difficult to enforce the required identities and to obtain a consistent representation in an optimal way. We do not show how to do it optimally here but we show that even a simple, and probably not very optimal, technique provided a meaningful reconstructions. This technique can be used to initialize a non-linear bundle adjustment. We believe that there is a good reason to hope that much better results would be obtained when employing better estimation techniques to cope with noise.

The principal difference of this paper to Triggs' factorization method on lines [13] is in representation of lines. Method [13] represents a line by a pair of points which are transfered from the first into other images using the epipolar geometries known from the trifocal tensors (so-called point transfer). The advantage of [13] is that both points and lines can be used. The price for not doing the point transfer in our method is payed by the necessity for enforcing the nonlinear identities.

The paper is structured as follows. In section 2, line representation by Plücker coordinates is adopted and the factorization based reconstruction from line correspondences in perspective images is formulated. The main idea of the approach is spelled out in section 3 and the two key components of the approach, the scale factors estimation and the enforcing the Klein identities, are described in more detail. Experimental results are shown in section 5 and further extensions are discussed in the concluding section.

2. Problem Formulation

Suppose a set of n 3D lines visible in m perspective images. The goal is to recover 3D structure (line locations) and motion (camera locations) from the image measurements. This recovery will be called *scene reconstruction*. No camera calibration or additional 3D information will be assumed, so it will be possible to reconstruct the scene up to a projective transformation of the 3D space.

Let $\mathbf{L}_l$ be the unknown Plücker line coordinates [2] of the 3D lines, $\mathtt{P}^i$ the unknown 3×4 camera projection matrices, and $\mathbf{l}_l^i$ the measured homogeneous coordinate vectors of the image lines, where $i = 1, .., m$ labels images and $l = 1, .., n$ labels lines. No point correspondences are used. An example of such line correspondences is shown in Figure 1.

Let $\mathbf{X}_p$ denote the homogeneous coordinate vectors of the 3D points and let $\mathbf{x}_p^i$ denote their projections into the images. The basic image projection equation says that $\mathbf{x}_p^i$ are the projections of $\mathbf{X}_p$ up to unknown scale factors λ_p^i:

$\lambda_p^i \mathbf{x}_p^i = \mathtt{P}^i \mathbf{X}_p$. A similar image projection equation holds for lines and says that $\mathbf{l}_l^i$ are the projections of $\mathbf{L}_l$ up to unknown scale factors γ_l^i

$$\gamma_l^i \mathbf{l}_l^i = \mathtt{Q}^i \mathbf{L}_l \tag{1}$$

where $\mathtt{Q}^i$ are the line projection 3×6 matrices [1] of rank 3 given by

$$\mathtt{Q}^i = \begin{bmatrix} \mathtt{P}^{i2} \wedge \mathtt{P}^{i3} \\ \mathtt{P}^{i3} \wedge \mathtt{P}^{i1} \\ \mathtt{P}^{i1} \wedge \mathtt{P}^{i2} \end{bmatrix} \tag{2}$$

where $\mathtt{P}^{ir\top}$ are the rows of the point camera matrix $\mathtt{P}^i$, and $\mathtt{P}^{ir} \wedge \mathtt{P}^{is}$ are the Plücker line coordinates of the intersection of the planes $\mathtt{P}^{ir}$ and $\mathtt{P}^{is}$ [2, page 187].

The complete set of image projections can be gathered into a matrix equation:

$$\underbrace{\begin{bmatrix} \gamma_1^1 \mathbf{l}_1^1 & \gamma_2^1 \mathbf{l}_2^1 & \dots & \gamma_n^1 \mathbf{l}_n^1 \\ \gamma_1^2 \mathbf{l}_1^2 & \gamma_2^2 \mathbf{l}_2^2 & \dots & \gamma_n^2 \mathbf{l}_n^2 \\ \vdots & & \ddots & \vdots \\ \gamma_1^m \mathbf{l}_1^m & \gamma_2^m \mathbf{l}_2^m & \dots & \gamma_n^m \mathbf{l}_n^m \end{bmatrix}}_{\mathtt{S}} = \underbrace{\begin{bmatrix} \mathtt{Q}^1 \\ \vdots \\ \mathtt{Q}^m \end{bmatrix}}_{\substack{\mathtt{Q} \\ 3m \times 6}} \underbrace{[\mathbf{L}_1 \dots \mathbf{L}_n]}_{\substack{\mathtt{L} \\ 6 \times n}}$$

where $\mathtt{L}$ and $\mathtt{Q}$ stand for structure and motion, respectively. The $3m \times n$ matrix $\mathtt{S}$ will be called the *rescaled line measurement matrix*. Only $\mathbf{l}_l^i$ are available from perspective images. Scalars γ_l^i are unknown. The task is to find scalars γ_l^i so that matrix $\mathtt{S}$ can be factorized into the matrices $\mathtt{Q} \in \mathbb{R}^{3m \times 6}$ and $\mathtt{L} \in \mathbb{R}^{6 \times n}$ such that every row of $\mathtt{Q}$ and every column of $\mathtt{L}$ as a vector, say $(v_1, v_2, v_3, v_4, v_5, v_6)^\top$, lies on the Klein quadric, i.e.

$$v_1 v_4 + v_2 v_5 + v_3 v_6 = 0 \tag{3}$$

which is the necessary condition for representing cameras and lines in Plücker coordinates.

3. The Main Idea of the New Factorization Algorithm

A factorization method requires that all elements of $\mathtt{S}$ matrix are known. Therefore, the scale factors in $\mathtt{S}$ have to be estimated before $\mathtt{S}$ can be factorized. The necessary condition for $\mathtt{S}$ being factorizable is that it is of rank six. However, it is not a sufficient condition and the matrices resulting from a plain factorization into a product of $\hat{\mathtt{Q}} \in \mathbb{R}^{3m \times 6}$ and $\hat{\mathtt{L}} \in \mathbb{R}^{6 \times n}$ by SVD as in eqs. (9) and (10) do not have to satisfy the Klein identities (3) on rows of $\hat{\mathtt{Q}}$ and columns of $\hat{\mathtt{L}}$, respectively. The situation can be remedied by finding a projection of the respective vectors onto the Klein quadric.

If $\mathtt{S}$ is a valid rescaled line measurement matrix, then $\mathtt{Q}$ and $\mathtt{L}$ satisfying (3) exist. Matrices $\mathtt{Q}$ and $\mathtt{L}$ can always be obtained from $\hat{\mathtt{Q}}$ and $\hat{\mathtt{L}}$ by some transformation $\mathtt{H} \in \mathbb{R}^{6 \times 6}$, $rank\, \mathtt{H} = 6$, as $\mathtt{Q} = \hat{\mathtt{Q}}\mathtt{H}$ and $\mathtt{L} = \mathtt{H}^{-1}\hat{\mathtt{L}}$ for each factorization pair is bound by a change of the basis

$$\mathtt{S} = \underbrace{\hat{\mathtt{Q}}\mathtt{H}}_{\mathtt{Q}} \underbrace{\mathtt{H}^{-1}\hat{\mathtt{L}}}_{\mathtt{L}} \tag{4}$$

1. Estimate the trifocal tensor $\mathcal{T}$ using line correspondences [2].
2. Compute proj. matrices $\mathtt{P}$, $\mathtt{P}'$, and $\mathtt{P}''$ using $\mathcal{T}$ [2, 14].
3. Compute proj. matrices $\mathtt{Q}$, $\mathtt{Q}'$, and $\mathtt{Q}''$ using eq. (2).
4. Compute $\mathbf{L}_l$ as intersections of the back-projected image lines [2] i.e. assemble the matrix $\mathtt{W}$

   $$\mathtt{W} = \begin{bmatrix} \mathbf{l}^\top \mathtt{P} \\ \mathbf{l}'^\top \mathtt{P}' \\ \mathbf{l}''^\top \mathtt{P}'' \end{bmatrix}$$

   and set $\mathbf{L}_l = \mathtt{v}(:,3) \vee \mathtt{v}(:,4)$ where $[\mathtt{u}, \mathtt{s}, \mathtt{v}] = \mathrm{SVD}(\mathtt{W})$ and $\mathbf{X}_1 \vee \mathbf{X}_2$ is a join of 3-space points $\mathbf{X}_1$ and $\mathbf{X}_2$ into the line in Plücker coordinates [2].
5. Estimate scale factors γ_l, γ_l', and γ_l'' according to

   $$\gamma_l^i = \frac{\bar{\mathbf{l}}_l^i \cdot \mathbf{l}_l^i}{\|\mathbf{l}_l^i\|^2} \tag{5}$$

   where $\bar{\mathbf{l}}_l^i$ are the projected lines $\mathbf{L}_l$ into the image i, $\bar{\mathbf{l}}_l^i = \mathtt{Q}^i \mathbf{L}_l$.

Algorithm 1: Scale factor estimation from a triple of views

3.1. Estimating the Scale Factors

The scale factor estimation is done by computing partial reconstructions from triples of images using trifocal tensors and re-projecting the reconstructions back into the images. Scale factors are computed from the difference between the reprojected and the original image lines. Eq. (5) is the best solution for γ_l^i in the least squares sense, which is a variation on eq. (3) in [10].[1] The whole algorithm for scale factor estimation is summarized in Algorithm 1.

Scale factors resulting from independent partial reconstructions from triples of views may be mutually inconsistent, which means that scale factors of a given image line, resulting from different triplets of views, differ. Therefore triples of views must be established so that they overlap by at least one view and in the sense that such overlaps form

[1] As pointed out by one of the reviewers, the scale factors can be estimated without explicit reconstruction, which is, however, needed in section 3.2, in the following way. Given consistent scales for the epipoles (in the sense of [10, 13, 12]), a consistent scaling for the image lines is $\gamma^i \mathbf{l}^i \cdot \mathbf{e}^{ij} = -\gamma^j \mathbf{l}^j \cdot \mathbf{e}^{ji}$ for any i, j.

one connected component. Rescaling eqs. (1) of all triplets of views can be then chained together for any given line l over m views by column rescaling to give a consistent $(\gamma_l^1, \gamma_l^2, \ldots, \gamma_l^m)^\top$ [9].

3.2. Enforcing the Klein quadric identities

The Klein quadric identities (3) on rows of $\mathtt{Q}$, resp. columns of $\mathtt{L}$, can be written in a matrix form as

$$diag(\mathtt{Q}\mathtt{E}_\mathtt{r}\mathtt{Q}^\top) = \mathtt{0}_{3m\times 1}, \text{ resp. } diag(\mathtt{L}^\top\mathtt{E}_\mathtt{r}\mathtt{L}) = \mathtt{0}_{n\times 1}, \quad (6)$$

where the 6×6 matrix $\mathtt{E}_\mathtt{r}$ is of the form

$$\mathtt{E}_\mathtt{r} = \begin{bmatrix} \mathtt{0}_{3\times 3} & \mathtt{I}_{3\times 3} \\ \mathtt{I}_{3\times 3} & \mathtt{0}_{3\times 3} \end{bmatrix}$$

and $\mathtt{I}_{3\times 3}$ is the identity matrix. Eqs. (6), after applying (4), become

$$\begin{aligned} diag(\hat{\mathtt{Q}}\mathtt{H}\mathtt{E}_\mathtt{r}\mathtt{H}^\top\hat{\mathtt{Q}}^\top) &= \mathtt{0}_{3m\times 1}, \text{ resp.} \\ diag(\hat{\mathtt{L}}^\top\mathtt{H}^{-\top}\mathtt{E}_\mathtt{r}\mathtt{H}^{-1}\hat{\mathtt{L}}) &= \mathtt{0}_{n\times 1}, \end{aligned} \quad (7)$$

which is quadratic in terms of elements of $\mathtt{H}$ and could be solved only nonlinearly. Fortunately, $\mathtt{H}$ can be found by another way. The following holds.

Proposition 1 *Let $\mathtt{S}$ be a rescaled line measurement matrix (no noise in data) composed from non-degenerate perspective images of lines in a scene taken by cameras in general positions. Let $\mathtt{S} = \hat{\mathtt{Q}}\hat{\mathtt{L}}$ be a plain factorization of $\mathtt{S}$ by* SVD *(not necessarily satisfying Klein identities (6)). Let*

$$\mathtt{S}_{9\times n} = \dot{\mathtt{Q}}_{9\times 6}\dot{\mathtt{L}}$$

be a partial reconstruction of the lines in the scene from a triple $\mathbf{t}$ of images obtained by Algorithm 1 (i.e. $\dot{\mathtt{Q}}_{9\times 6}$ and $\dot{\mathtt{L}}$ satisfy the Klein identities (6)). If matrix $\mathtt{H}$ satisfies

$$\hat{\mathtt{Q}}_{9\times 6}\mathtt{H} = \dot{\mathtt{Q}}_{9\times 6} \quad (8)$$

then $\mathtt{Q} = \hat{\mathtt{Q}}\mathtt{H}$ as well as $\mathtt{L} = \mathtt{H}^{-1}\hat{\mathtt{L}}$ satisfy the Klein identities (6).

Proof: Columns of $\dot{\mathtt{L}}$ satisfy Klein identities since they are equal to coordinates of space lines reconstructed from a triple of images via a trifocal tensor. $\mathtt{L} = \mathtt{H}^{-1}\hat{\mathtt{L}} = \dot{\mathtt{L}}$ thus satisfies Klein identities also. In general, lines of $\dot{\mathtt{L}}$ are linearly independent. Therefore, there is exactly one $\dot{\mathtt{Q}}$ of coefficients that linearly combines the rows of $\dot{\mathtt{L}}$ into $\mathtt{S}$ as $\mathtt{S} = \dot{\mathtt{Q}}\dot{\mathtt{L}}$. Matrices $\hat{\mathtt{Q}}_{9\times 6}$, $\dot{\mathtt{Q}}_{9\times 6}$ are in general of rank six and thus (8) fixes $\mathtt{H}$ uniquely. Since there is exactly one $\dot{\mathtt{Q}}$ so that $\mathtt{S} = \dot{\mathtt{Q}}\dot{\mathtt{L}}$ the linear mapping bringing $\hat{\mathtt{Q}}$ into $\mathtt{Q}$ (which exists by the argument given at the next paragraph) equals $\mathtt{H}$. □

When there is noise in the data, $\mathtt{H}$ obtained according to eq. (8) does not have to fulfill the Klein identities (7). Nonlinear bundle adjustment with initial solution $\mathtt{H}$ should be

1. Establish triplets of views among m views such that the triplets overlap as explained in section 3.1. For each triplet of views, compute the scale factors using Algorithm 1.

2. Chain the rescaling equations (1) of all triplets of views together for any given line l over m views to give a consistent $(\gamma_l^1, \gamma_l^2, \ldots, \gamma_l^m)^\top$. Denote the triple whose scale factors have not been changed during the rescaling by $\mathbf{t}$.

3. Factorize complete rescaled line measurement matrix $\mathtt{S} = \left[\ \gamma_l^i \mathbf{l}_l^i\ \right]_{i=1..m, l=1..n}$ into matrices $\hat{\mathtt{Q}}$ and $\hat{\mathtt{L}}$ as

$$\begin{aligned} \hat{\mathtt{Q}} &= \mathtt{u}(:, 1:6) & (9) \\ \hat{\mathtt{L}} &= \mathtt{s}(1:6, 1:6)\ \mathtt{v}(:, 1:6)^\top & (10) \end{aligned}$$

   where $[\mathtt{u}, \mathtt{s}, \mathtt{v}] = \text{SVD}(\mathtt{S})$.

4. Find the transformation matrix $\mathtt{H}$ transforming the rows of $\hat{\mathtt{Q}}$ corresponding to triple $\mathbf{t}$ into the basis of the partial reconstruction of triple $\mathbf{t}$, i.e.

$$\begin{bmatrix} \hat{\mathtt{Q}}^i \\ \hat{\mathtt{Q}}^j \\ \hat{\mathtt{Q}}^k \end{bmatrix} \mathtt{H} = \begin{bmatrix} \mathtt{Q} \\ \mathtt{Q}' \\ \mathtt{Q}'' \end{bmatrix}$$

   where $\mathtt{Q}$, $\mathtt{Q}'$, and $\mathtt{Q}''$ come from scale factor estimation for triple $\mathbf{t} = (i, j, k)^\top$, step 3 in Algorithm 1.[a]

5. Apply transformation $\mathtt{H}$ so that the result is close to the Klein quadric: $\tilde{\mathtt{Q}} = \hat{\mathtt{Q}}\mathtt{H}$, $\tilde{\mathtt{L}} = \mathtt{H}^{-1}\hat{\mathtt{L}}$.

6. Project rows of $\tilde{\mathtt{Q}}$ and columns of $\tilde{\mathtt{L}}$ onto the Klein quadric as in sec. 3.2 to gain $\mathtt{Q}$ and $\mathtt{L}$, respectively.

[a] An alternative way is to find $\mathtt{H}$ as $\mathtt{H} = \mathtt{G}^{-1}$ where $\hat{\mathtt{L}}^\top\mathtt{G} = \mathtt{L}^{\mathbf{t}\top}$, which appeared to be less sensitive to noise.

Algorithm 2: Scene reconstruction from lines

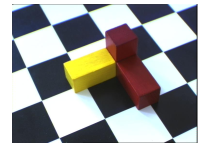
used. Another, but only approximate, solution is obtained by finding some linear projections of each row of $\hat{\mathtt{Q}}$, resp. column of $\hat{\mathtt{L}}$, onto the Klein quadric. If there is noise in the data, matrix $\mathtt{H}$ from Proposition 1 does not have to exist. However, it is always possible to find $\mathtt{H}$ using a partial reconstruction from three views so that eq. (8) holds. It turned out in our experiments (see sec. 5) that although rows of $\hat{\mathtt{Q}}\mathtt{H}$, resp. columns of $\mathtt{H}^{-1}\hat{\mathtt{L}}$, do no satisfy the Klein identities, they are close to the Klein quadric so that a good solution can be obtained by projecting them onto the Klein quadric.

Projection onto the Klein quadric was done in the following way. For each view, i, system of linear equations

$$\mathbf{l}_l^{i\top}\mathtt{P}^i\tilde{\mathbf{X}}_{l,p} = 0 \text{ for } l = 1..n,\ p = 1, 2 \quad (11)$$

was used to estimate point projection matrix $\mathtt{P}^i$ from image measurements and matrix $\tilde{\mathtt{L}} = \mathtt{H}^{-1}\hat{\mathtt{L}}$ where $\tilde{\mathbf{X}}_{l,p} \in \mathbb{R}^4$ are

Scene *Cubes*	5 images [576×768], 14 correspondences, manual detection		
Method $\setminus$ γ_l^i estimation	*sequence, two view overlap*	*two central images*	*no factorization*
Factorization	8.69 / 155.14 / 2.46	6.03 / 74.24 / 2.81	
Linear method	**3.80** / 24.03 / 2.45	**2.42** / 13.41 / 1.66	**1.89** / 11.09 / 1.08
Linear method + BA	**0.47** / 2.12 / 0.34	**0.47** / 2.12 / 0.33	**0.47** / 2.12 / 0.33



Scene *House (Oxford)*	6 images [576×768], 31 correspondences, automatic detection		
Method $\setminus$ γ_l^i estimation	*sequence, two view overlap*	*two central images*	*no factorization*
Factorization	4.62 / 42.98 / 2.40	3.38 / 30.17 / 1.91	
Linear method	**1.57** / 23.24 / 0.77	**0.80** / 10.53 / 0.44	**1.03** / 13.33 / 0.52
Linear method + BA	**0.23** / 1.32 / 0.17	**0.23** / 1.32 / 0.17	**0.23** / 1.32 / 0.17

Figure 2: *Experiments with real scenes.* Mean / maximal / median reprojection errors are shown

some two columns of the dual Plücker matrix of $\tilde{\mathbf{L}}_l$. Finally, each line on the Klein quadric was estimated by intersecting backprojections of the image measurements using all point projection matrices as in step 4 of Alg. 1. Our new method for scene reconstruction from lines is summarized in Alg. 2.

4. Implementation Details

On account of good numerical conditioning, several normalizations of the data and balancing similar to those in [10] need to be performed.

5. Experiments

The new method has been tested on a simulated scene and on two real scenes. No point correspondences have been used for the line reconstruction by Algorithm 2. The reconstructed 3-space lines were reprojected into the images. The reprojection error of a line was computed as the mean of Euclidean distances between the end-points of the original image line and the reprojected reconstructed line.

An artificial scene was used for an experiment with simulated data. 20 images of 30 lines in space have been obtained from different viewpoints. The reconstruction was precise in absence of noise and the mean error of the reconstruction grew linearly with the variance of the added noise.

The tables describing each real experiment in Figure 2 include scene name, number of images and their sizes, number of correspondences together with the way of their detection. The reconstruction method was used in two setups of image triplets: (i) sequence with two view overlap and (ii) two central images. For each setup, reprojection errors of the plain factorization into $\hat{\mathtt{Q}}\hat{\mathtt{L}}$, of the linear method, and of the non-linear line bundle adjustment initiated by the linear method are shown. For comparison, the following simple reconstruction method without factorization is given. (i) $\mathtt{L}$ from a partial reconstruction of an image triple was used to estimate point projection matrices using eq. (11). Then, (ii) backprojections from all images were intersected as described in section 3.2.

In the Cube scene, five images of cubes on a checkerboard have been obtained from different viewpoints, three of them can be seen in Fig. 1. 14 correspondences of lines, at least partially visible in all images, have been detected manually. Mean, resp. maximal, reprojection errors of the linear method of the better setup were 2.42, resp. 13.41, pixels. The House scene was obtained on six images, see Fig. 3. The original lines are drawn in white and the reprojected ones in black color. Mean, resp. maximal, reprojection errors of the linear method of the better setup were 0.80, resp. 10.53, pixels.

In both experiments with real scenes, the way of scale factor estimation with two central images provided a better solution. Both factorizations of the Cube scene provided worse solutions than the reconstruction without factorization. This may be due to a little amount of (14) correspondences and unprecise manual line detection. On the other hand, 31 automatic correspondences in the House scene enabled improvement of the factorization by 20% compared to the simple method.

6. Summary and Conclusions

A new linear method for line reconstruction via a factorization of a line measurement matrix has been proposed and tested on simulated and real scenes. The method is, in principle, capable to reconstruct lines even though there are no corresponding points on the lines available. We have used line projection by a perspective camera to formulate the reconstruction as a factorization and showed how to carry it out in a noise free situation.

We have pointed out that finding the optimal reconstruction w.r.t. noise in data is, as usual, a non-linear task but demonstrated that the vectors obtained by plain SVD factorization followed by a transformation considering the constraint (3) provide a good approximate solution that may be hoped as a starting point for nonlinear bundle adjustment.

The line factorization method can be straightforwardly extended to deal with occluded lines using Jacobs' algorithm [4] as it was performed in [8]. There exist many ways

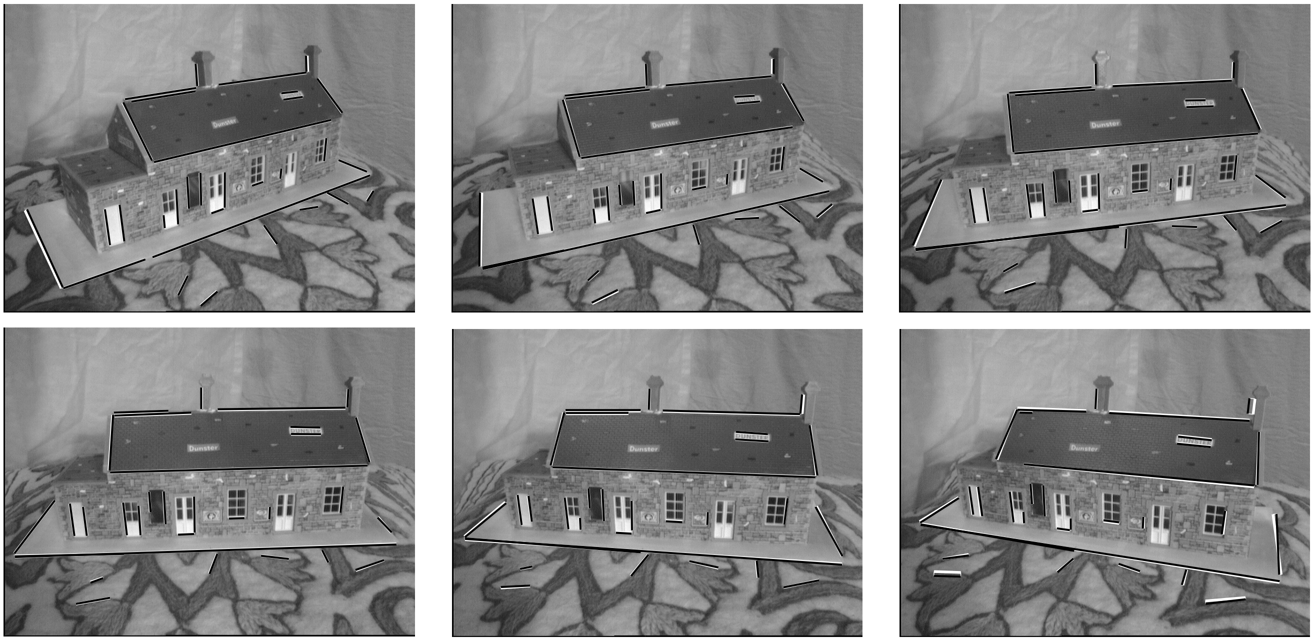


Figure 3: *Line reconstruction from many images.* No point correspondences have been used for the line reconstruction by Algorithm 2. Mean / maximal reprojection errors of the method without bundle adjustment were 0.80 / 10.53 pxl

how to establish triplets of views in step 1 of Algorithm 2. In presence of noise, reconstructions resulting from different sets of triplets differ in the reprojection error. Similar heuristics for choosing the best set of triplets based on the structure of the missing data to those in [8] can be used.

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